Statistical and Low Temperature Physics (PHYS393)

# 5. Matter at Low Temperatures

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Matters at low temperatures can exist in phases that are different from what we know at room temperature. They can have properties that are very different from what we know in everyday life.

We shall focus on the following:

1. Liquid helium becoming a superfluid.

2. Metals becoming superconducting.

When helium-4 is cooled below 2.17 K, it becomes superfluid.

This is a phase of matter that is different from any liquid at room temperature - it has no viscosity at all.

When a stone is dropped in water, it experiences resistance to its movement because of viscosity. When the same stone is dropped in superfluid helium-4, it experiences no resistance at all.

Consider a piece of wire made of tin. When it is cooled below 3.7 K, it becomes superconducting.

This is a property that is different from any electrical conductivity at romm temperature - the wire has no electrical resistance at all.

At room temperature, an electric current flowing through a metal would always experience some resistance. For some metals, like tin, this resistance vanishes below a certain temperature (usually a few Kelvins).

In order to understand these behaviours, we study the following:

1. why helium remains liquid to 0 K,

2. what zero resistance means for excitations (e.g. phonons),

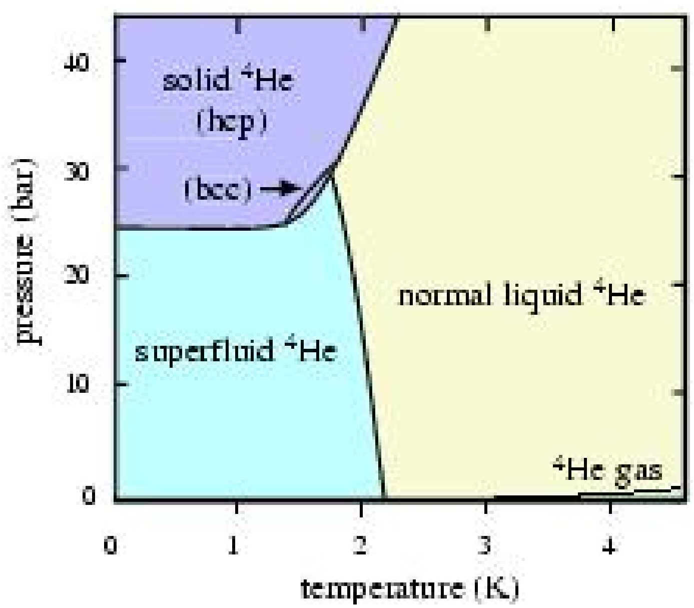
3. what the heat capacity can say about excitations,

4. how the wavefunction also has zero resistance, and

5. the idea of a macroscopic (giant) wavefunction.

When temperature falls, we expect matter to change from gas to liquid to solid. Helium-4, however, remains a liquid right down to 0 K - another unusual property.

The phase diagram shows that helium can only become a solid if the pressure is higher than 25 atm.




http://ltl.tkk.fi/wiki/LT/%C2%B5KI_Group/Helium_Crystals

Consider an atom in liquid helium-4. It lives in a small volume, surrounded by other atoms.

We can think of the atom as being confined to a small space of size $x$. According to Heisenberg's uncertainty principle, $xp \geq \hbar/2$, where $p$ is the momentum.

This corresponds to an energy $E = p^2/2m$, where $m$ is the mass of the atom. This is the zero point energy, which the atom would retain even at 0 K.

In the case of helium-4, this energy is greater than the binding energy from the attraction of the neighbouring atoms.

This is why helium-4 remains liquid right down to 0 K.

One simple way to start to understand superfluid is to think about excitations.

The exctitation here refers to exciting the fluid, or particles in the fluid, to a higher energy state. Imagine a large body moving through a liquid. It collides with the particles in the fluid.

If the collision is too weak, it may not cause any excitation, since energy levels occur in discrete steps (i.e. they are quantised).

This suggests that there must be a velocity of the body below which there would be no excitation.

No excitation means no viscosity. Why?

If there is resistance, the body must give up some energy. By energy conservation, this can only go to create excitations.

To develop the excitation idea further, lets recall the concept of the dispersion relation. This is the relation between energy and momentum of a particle. (It can also be expressed in terms of frequency and wavevector.)

For example, the dispersion relation for a particle of mass $m$ in free space with energy $E$ and momentum $p$ is $E = p^2/2m$.

Or, if the particle is a photon, then the relation is $E = pc$, where $c$ here is the speed of sound.

We shall make use of the idea of an elementary excitation.

For instance, to produce photons with momentum $p$, we must have at least $E = pc$, which makes one photon. If we have less than $E$, no photon is produced. Thus the photon is an elementary excitation.

Return to the idea of a large body moving through a superfluid. We can derive an expression for the velocity that would produce an elementary excitation.

Consider a large body of mass $M$ moving at velocity $\mathbf{v}$. Suppose it collides with a particle strongly enough to produce an elementary excitation of energy $E$.

After that, it's velocity changes to $\mathbf{v}'$. By energy conservation,

$$\frac{1}{2}Mv^2 = \frac{1}{2}Mv'^2 + E.$$

Suppose that the momentum of the excitation is $\mathbf{p}$. By momentum conservation,

$$M\mathbf{v} = M\mathbf{v}' + \mathbf{p}.$$

Rewrite the equations to give:

$$(v+v')(v-v') = \frac{2E}{M}$$

and

$$\mathbf{v}-\mathbf{v}' = \frac{\mathbf{p}}{M}.$$

From these, we can get

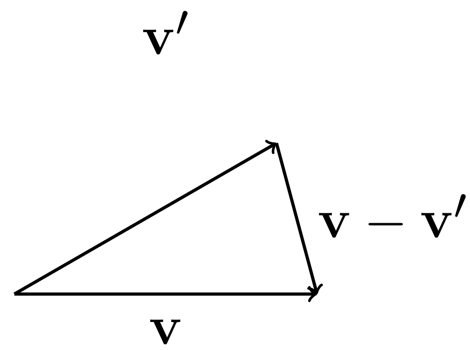
$$2v(v-v') \approx \frac{2E}{M}$$

and

$$|\mathbf{v}-\mathbf{v}'| = \frac{p}{M},$$

where we have used $v' \approx v$ because the mass of the body is much larger than that of the excitation particle.

From this vector diagram




we can see that

$$|\mathbf{v} - \mathbf{v}'| \geq v - v',$$

because $v - v'$ is nearly zero, since $v' \approx v$.

Rewriting the equations in this form,

$$(v - v') \approx \frac{E}{vM}$$

and

$$|\mathbf{v} - \mathbf{v}'| = \frac{p}{M},$$

and substituting into

$$|\mathbf{v} - \mathbf{v}'| \geq v - v',$$

we find

$$\frac{p}{M} \geq \frac{E}{vM},$$

or

$$v \geq \frac{E}{p}.$$

So $v$ must be larger than $E/p$ in order to produce any excitation.

Recall that $E$ and $p$ must be related by the dispersion relation.

Suppose the minimum value of $E/p$ is not zero. Let this be

$$v_L = \left(\frac{E}{p}\right)_{min}.$$

We have shown that $v$ must be more than this in order to produce any excitation.

So if the body moves at a velocity below $v_L$, it would not produce any excitation. This means it would not experience any viscosity - i.e. the fluid is a superfluid.

$v_L$ is called the Landau critical velocity.

Conversely, if the smallest $E/p$ in a fluid is not zero, then superfluidity is possible. This is because if the body moves at a velocity below $(E/p)_{min}$, it experiences no viscosity.

Let us now look at the typical dispersion relations in matter and consider if or how this is possible.

Two common ways in which the moving body can produce excitations are:

1. deflecting a helium atom, and
2. creating a phonon (particle of wave motion).

The dispersion relation for an atom is

$$E = \frac{p^2}{2m}.$$

So

$$\frac{E}{p} = \frac{p}{2m}.$$

Therefore, the smallest $E/p$ is zero - when $p = 0$.

Therefore, the Landau critical velocity $v_L = 0$. This means that there can be no superfluid.

The dispersion relation for a phonon is

$$E = pc.$$

So

$$\frac{E}{p} = c.$$

This means that $E/p$ is constant. Therefore, the smallest $E/p$ is $v_L$.

Therefore, the Landau critical velocity $v_L = c$. This means that the fluid would be superfluid, as long as the body does not move faster than the speed of sound.

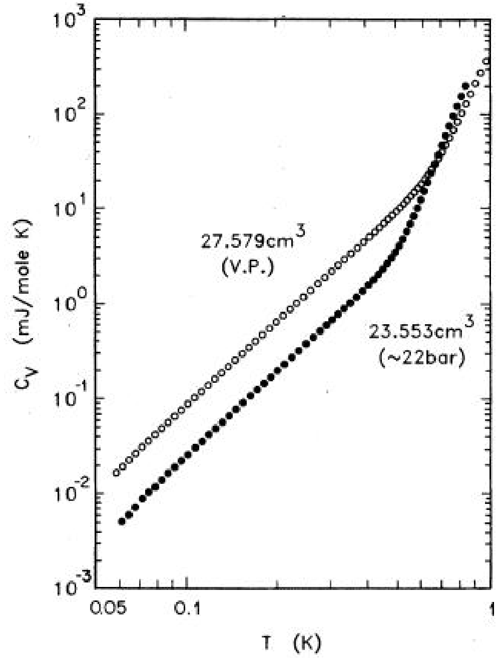
Lets see if how this can help explain why helium-4 can be a superfluid:

1. It is easy to understand why a moving body can excite phonons, since this is simply wave motion.

2. It is difficult to understand why the body cannot deflect helium atoms in the superfluid.

We shall return to the issue about deflecting atoms later on. For now, lets pursue the phonon idea further.

It is possible to verify that phonons are indeed the main contribution to heat capacity of liquid helium-4 at low temperature T.




D.S. Greywall, Phys. Rev. B 18, 2127 (1978)

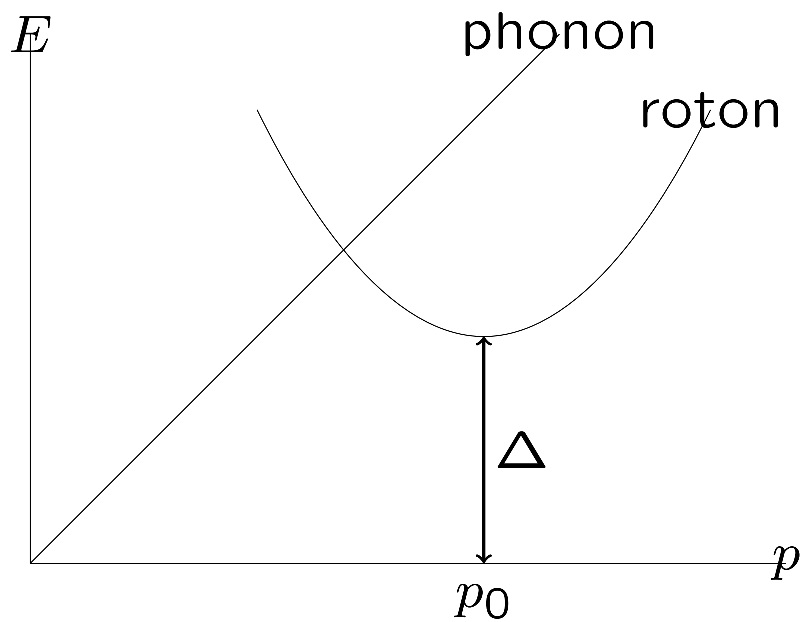
The measured heat capacity varies as $T^3$ below 1 K. This is consistent with the phonon contribution.

Above 1 K, the heat capacity increases faster than $T^3$.

In 1947, Landau, a Russian physicist, suggested that this is due to a new type of excitation that is different from phonon. He thought it has something to do with rotation of the liquid, so he called it "roton".

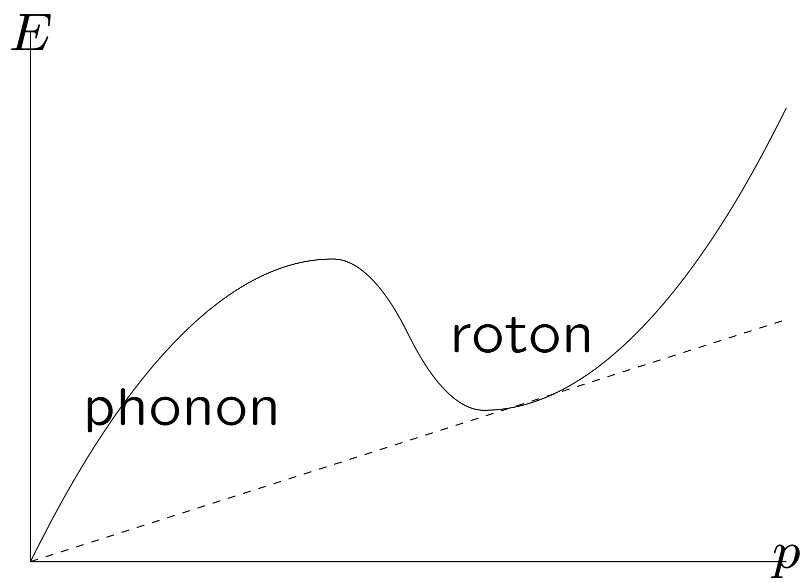
The rotons would coexist with phonons, but would require at least an energy $\Delta$ to create.

Landau suggested that these rotons follow a dispersion relation like the parabola in this figure:




By adjusting the size and shape of this parabola, Landau was able to calculate the heat capacity to fit the experiments.

Since we are only interested in the smallest excitation for at each momentum $p$, Landau's dispersion relation roughly looks like this:



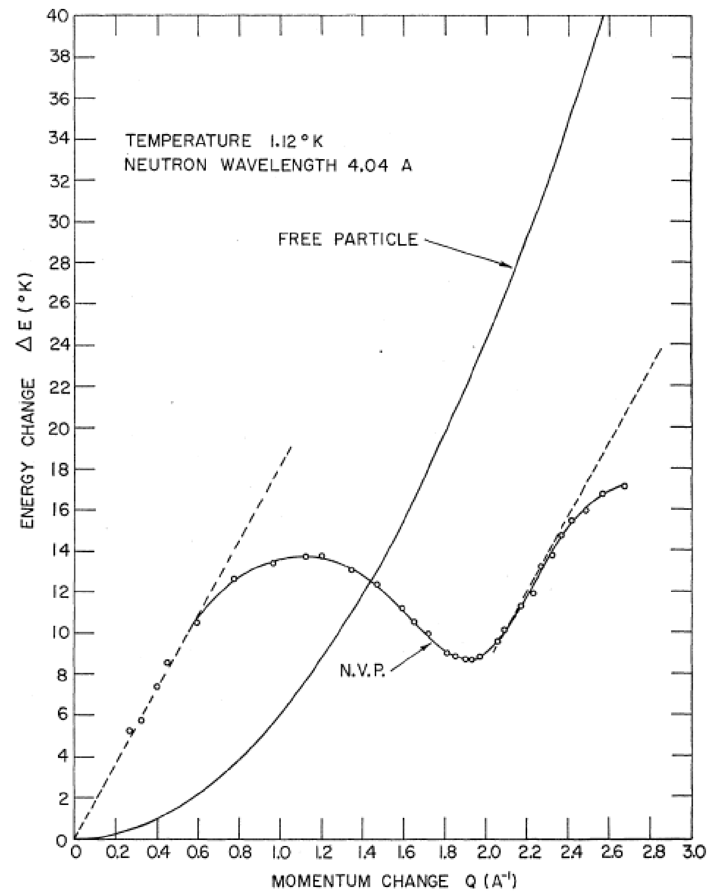

The minimum $E/p$ is now given by the gradient of the dashed line.

Therefore, the critical velocity is smaller than if only phonons were present.

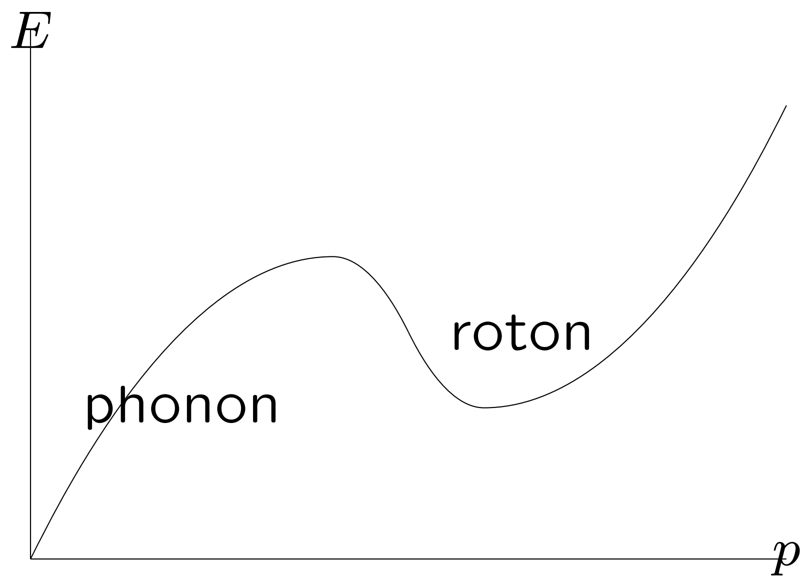
When Landau proposed his rotons, the only justification was that the predicted heat capacity agreed with measured values. One could argue that there might be other reasons why the heat capacity deviated from $T^3$.

Not long after, experiments using neutrons were used to measure the momenta of the excitations in liquid helium-4. By measuring the change in momenta of the scattered neutrons, we could deduce the momenta given up to the excitations.

In 1961, Landau's prediction of the dispersion relation is fully confirmed by neutron scattering experiments.




D. G. Henshaw and A. D. B. Woods, Phys. Rev. 121, 1266 (1961).



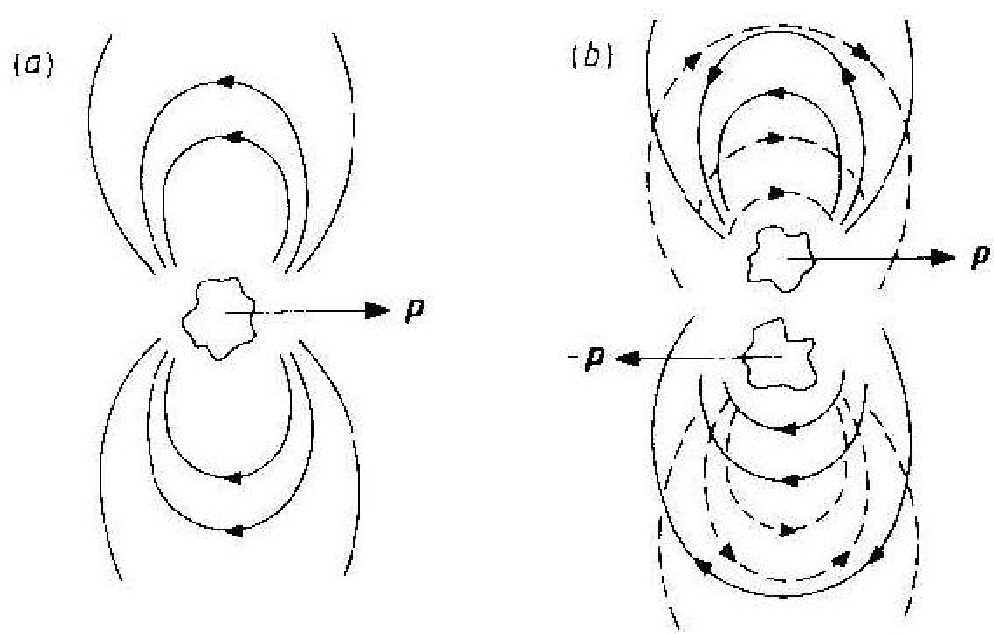

Although the dispersion relation is modified in this way, our earlier ideas about the Landau critical velocity remains valid.

In this new dispersion relation, the presence of the minimum due to rotons lowers $(E/p)_{min}$, but not to zero.

So helium-4 can be superfluid for a body moving below the critical velocity.

It should be mentioned that although Landau's dispersion relation has been verified, the nature of rotons is still unclear.

Rotons have been observed by neutron scattering experiments. Theories developed by Richard Feynman and others suggest that they are some kind of rotation.



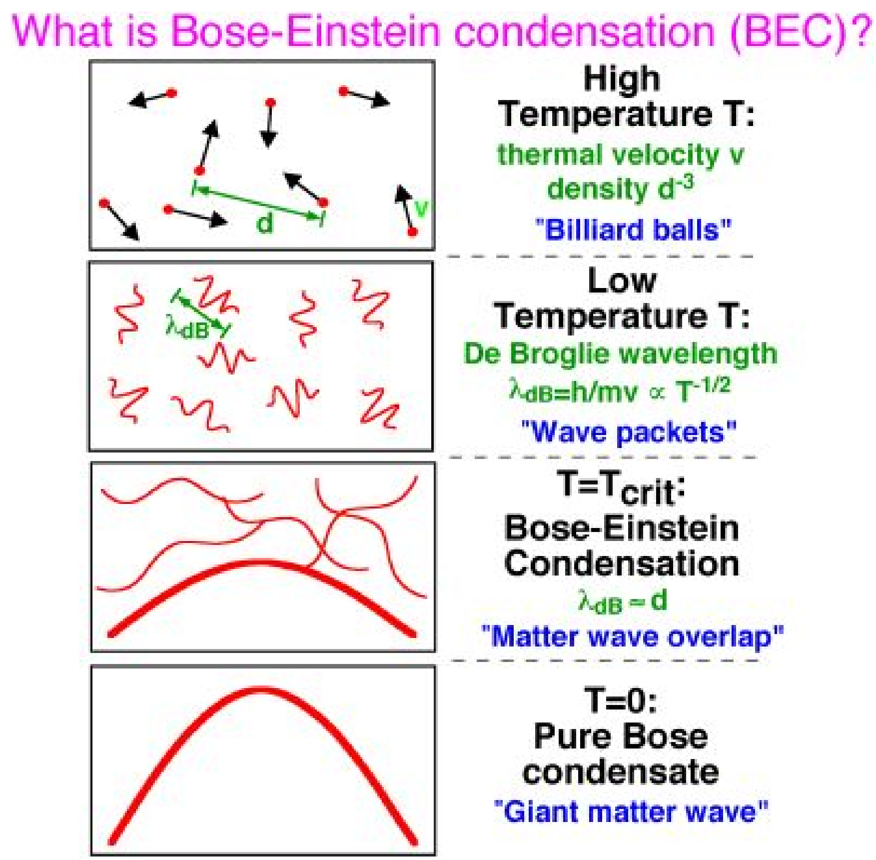

(Greytak, Quantum Liquids, Ruvalds and T Regge eds., (1978), p. 121.)

We have seen how superfluidity can happen if phonons and rotons are the only excitations. Let us now consider why excitations do not take place through direct collision of the moving body with the helium atoms.

Around 1938, Fritz London, a German physicist, suggested that superfluidity is related to Bose-Einstein condensation.

The idea is that all of the helium-4 atoms in the liquid somehow form a single, massive wavefunction.

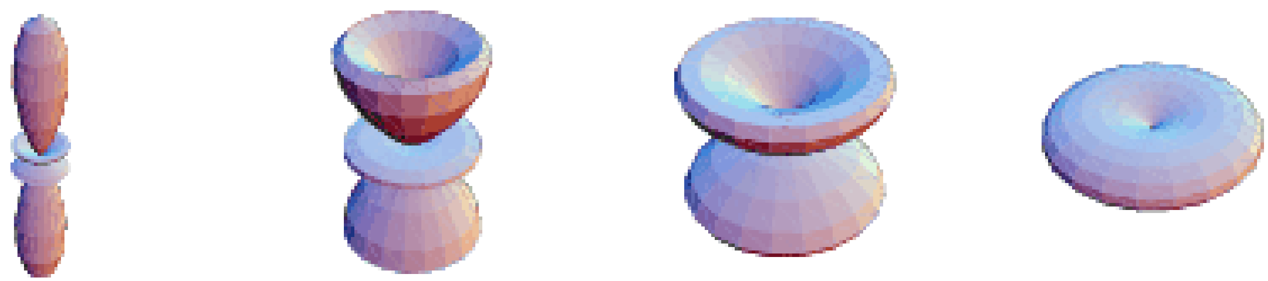
# Macroscopic wavefunction.




`http://cua.mit.edu/ketterle_group/intro/whatbec/whtisbec.html`

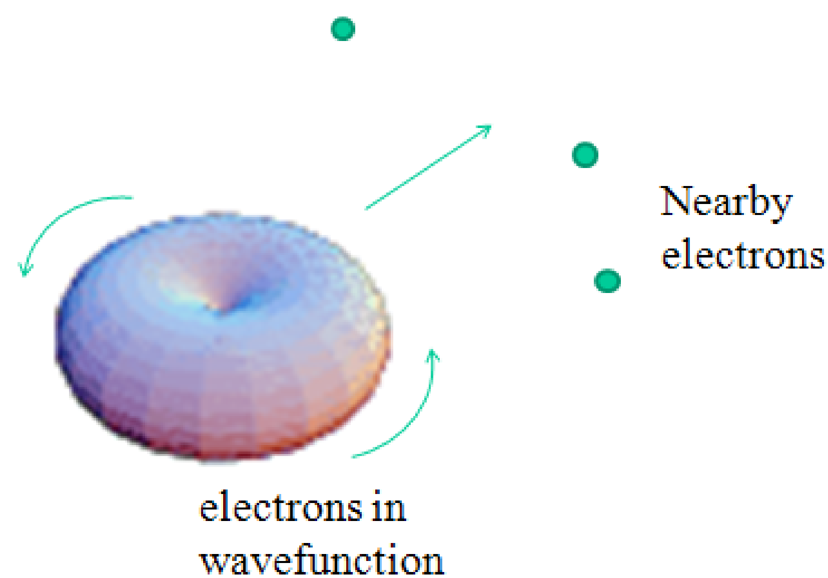
How can the idea of a be related to dissipation (friction or viscosity)?

The following figures are often used to represent the wavefunctions of electrons orbiting an atom.



http://www.physics.umd.edu/courses/Phys402/AnlageSpring09/Supp.htm

Consider electrons in a particular wavefunction in an atom. The electrons are circulating the atom, and the atom is moving through a cloud of very low energy electrons.




When the atom meets the nearby electrons, the random forces that the electrons in the atom would experience is like friction.

However, unless the nearby electrons have enough energy to excite the electrons in the atom to the next energy level, nothing would happen.

So the electrons in the atom can experience all the “friction” and still keep their own energies and momenta.

This idea can be applied to superfluid helium.

1. Think of the helium atoms superfluid like electrons in an atomic wave function.

2. Think of a body in the superfluid as a nearby electron.

3. If the superfluid wavefunction flows past the body slowly, there is not enough energy to excite the helium atoms to the next energy level.

4. So the helium atoms keep the same energy. There is no loss of energy, so no resistance is experienced.

5. So as long as the liquid helium flows slowly enough, it would be superfluid.

Although the reasoning is logical, it is important to realise that we are not talking about a wavefunction in an atom.

We are speaking of a wavefunction that can be as big as a bucket of water!

This is a big leap of the imagination, first taken by Fritz London in 1938. Since than, many many experimental evidence have been accumulated to support this idea that the superfluid is indeed a macroscopic wavefunction.

Fritz London's suggestion in 1938 was that the helium-4 superfluid is a Bose Einstein condensate (BEC).

Being a BEC means that all of the helium atoms would belong to a single wavefunction at the ground state. A body moving through it would not experience any viscosity as long as there is not enough energy to excite the wavefunction to the next energy level.

It turns out that the same ideas used to explain superfluidity can also be used to explain why electrons can flow without resistance in a superconductor.

In the next few lectures, we shall see how these ideas can be further developed and tested by experiments.

# Useful readings.

Fritz London - BEC model
L. Tisza - 2 fluid model
http://www.nature.com/physics/looking-back/superfluid3/index.html

fountain effect
http://www.nature.com/physics/looking-back/superfluid2/index.html

superfluid - kapitza
http://www.nature.com/physics/looking-back/superfluid/index.html

Superfluidity - Leggett
Rev. Mod. Phys. 71, S318-S323 (1999)

Superfluidity - R. P. Feynman and Michael Cohen
Phys. Rev. 102, 1189-1204 (1956) - section 1 and 2

superconductivity - f. london
http://www.nature.com/physics/looking-back/london/index.html

Superconductivity above 130 K
http://www.nature.com/physics/looking-back/schilling/index.html

what is BEC400.jpg - Ketterle
http://cua.mit.edu/ketterle_group/intro/whatbec/whtisbec.html